Dynamic Systems

An important aspect in mechatronic systems is the dynamic behavior, i.e. the behavior of the system as function of time. Especially in systems that display swift changes or systems that should behave accurately, the dynamic behavior is important. It is therefore useful to predict the dynamic behavior of a system. Modeling and simulation is useful for making such predictions.

There are various methods of modeling and simulating dynamic systems. A well known one is the Lumped Parameter Method.

With the Lumped Parameter Method, the dynamic behavior of a system is concentrated in discrete points. The interaction of these points gives us insight in the behavior of the real system. The more discrete points are used the more accurate the model will be.

There are various ways to represent lumped parameter models. Well known representations are iconic diagrams, differential equations, block diagrams and bond graphs.

Let's consider the suspension of a car. A lumped parameter model starts with the identification of the various lumps (or parts or components) of this system. We start with the car body. It is supposed to be a rigid body and therefore this part is represented with the icon of a mass. The suspension of the car is represented by a spring damper combination. The wheel is considered to have a significant mass and to be elastic. This component is therefore represented by a mass and spring icon. Finally the road is modeled by path generator function. The resulting ideal physical model (IPM) is shown below.

Iconic diagram of a car suspension.
**Bond Graphs**

Bond graphs are a network-like description of physical systems in terms of ideal physical processes. With the bond graph method we split up the system characteristics into an (imaginary) set of separate elements. Each element describes an idealized physical process. To facilitate drawing of bond graphs, the common elements are denoted by special symbols.

Look again at the car suspension example. In the picture below at the right a bond graph is shown that has been entered in 20-sim. All elements of the ideal physical model have corresponding elements in the bond graph. The connections between the elements in the bond graph, which are known as bonds, represent ideal energy transfer between the elements, i.e. no energy is stored, generated or dissipated. Bonds are drawn with harpoons (the half arrows).

*The car suspension model (middle) and corresponding bond graph model (right).*

For the mechanical domain, ideal velocity sources are in bond graphs denoted by the symbol $Sf$. Dampers are denoted by an $R$, springs by a $C$ and masses by a $I$. With a $1$ a structural connection of elements is denoted and with a $0$ a velocity difference is denoted.

A bond graph describes a physical system as a number of physical concepts (the elements) connected by energy flows (the bonds).
**Effort and Flow**

A bond between two elements transfers power from one element to the other. This flow of energy can be described in many ways. For bond graphs a uniform approach is chosen:

The flow of energy between two elements (and thus a bond) is always characterized by two variables, of which the product is power. According to the bond graph notation, these variables are called **effort** \((e)\) and **flow** \((f)\).

\[ \text{element} \quad e \quad \rightarrow \quad f \quad \rightarrow \text{element} \]

*Effort and flow as variables of a bond.*

The effort and flow variables make up a combination that is typical for a physical domain. The product of effort and flow is always power. We call such a pair of variables **power conjugated variables**. For example voltage and current are used for electrical networks and force and velocity are used for mechanical (translation) systems. The table below shows the variables for the domains that are currently supported in 20-sim.

<table>
<thead>
<tr>
<th>Domain</th>
<th>effort ((e))</th>
<th>flow ((f))</th>
</tr>
</thead>
<tbody>
<tr>
<td>power</td>
<td>effort (e)</td>
<td>flow (f)</td>
</tr>
<tr>
<td>mechanical (translation)</td>
<td>force (F) [N]</td>
<td>velocity (v) [m/s]</td>
</tr>
<tr>
<td>mechanical (rotation)</td>
<td>torque (T) [Nm]</td>
<td>angular velocity (omega) [rad/s]</td>
</tr>
<tr>
<td>pneumatic</td>
<td>pressure (p) [Pa]</td>
<td>volume flow (phi) [m(^3)/s]</td>
</tr>
<tr>
<td>thermal</td>
<td>temperature (T) [K]</td>
<td>entropy flow (dS) [J/Ks]</td>
</tr>
<tr>
<td>electric</td>
<td>voltage (u) [V]</td>
<td>current (i) [A]</td>
</tr>
<tr>
<td>hydraulic</td>
<td>pressure (p) [Pa]</td>
<td>volume flow (phi) [m(^3)/s]</td>
</tr>
<tr>
<td>magnetic</td>
<td>current (i) [A]</td>
<td>voltage (u) [V]</td>
</tr>
<tr>
<td>pseudothermal</td>
<td>temperature (T) [K]</td>
<td>heat flow (dQ) [W]</td>
</tr>
</tbody>
</table>

*The effort and flow variables for several domains.*

To most general domain is power. Bonds of this domain can connected to elements of all domains. For the thermal domain there is one pair of effort and flow, \(T\) and \(dS\), that multiplies to power. The pair \(T\) and \(dQ\) however, is more often used but does not multiply to power. Therefore the domain with these variables is called pseudothermal.

**Note**

- There is a direct relation between the across and through variables of an iconic diagram an the effort and flow variables of a bond graph:

\[
\begin{array}{ccc}
\text{Domain} & \text{Non-mechanical Domains} & \text{Mechanical Domains} \\
\hline
\text{across} & \text{effort} & \text{flow} \\
\end{array}
\]
The relation between across and through variables and effort and flow variables.

- 20-sim propagates domains. If a bond of the general power domain is coupled with an element of the electrical domain, it automatically becomes an electrical bond. The other end of the bond can then only be connected to another element of the electrical domain.
- If you want to use a bond of a domain that is not supported or has variables other than across and through, use the general power domain. The across and through variables can then be used as an alias for your own variables.
Bonds

The power of a bond is positive when both effort and flow have a positive value or both effort and flow have a negative value (power = effort × flow).

To denote the orientation, i.e. the direction of positive power, we use the harpoon at the end of a bond. An element with an inward bond connected, consumes power when the product of effort and flow is positive.

![A bond as power connection.](image)

When two elements are connected by a bond, one element will always determine the effort, while the opposite element will always determine the flow. The element that determines the effort, gets an enforced flow from the other element.

We can therefore also see a bond as a bilateral signal connection (effort-signal and flow-signal), of which the directions are opposite to each other.

With *direction* we mean the direction of the flow of information, just like in a block diagram. In a block diagram the effort and flow variables, which together form the flow of power, are not shown as a couple. This breaks up the symmetry between the physical system and the the structure of the model.

![A bond as a bilateral signal flow.](image)

The interpretation of a bond as a bilateral signal flow, does not fix the individual direction of the effort and the flow. It only means the direction of the effort and flow are opposite. For the derivation of a simulation model out of a bond graph however, the individual directions are of importance.

*To indicate the individual direction of the effort and flow, we use a small stroke (causal stroke) perpendicular to the bond. This stroke indicates the direction of the effort. The direction of the flow is opposite.*

The choice of the direction of signals, also known as causality, depends on the element that is connected to the bond.
Standard Elements

To facilitate drawing of bond graphs, the common elements are denoted by special mnemonic symbols. The elements can be divided in several classes:

- **Junctions**: (0 junction, 1 junction) elements that couple energy between various other elements.
- **Buffers**: (C,I) elements that store energy.
- **Dissipators**: (R) elements that dissipate energy.
- **Sources**: (Se,Sf) elements that generate energy.
- **Modulated Sources**: (MSe, MSf) elements that generate energy (driven by signals).
- **Transformers and Gyrators**: (TF, GY) elements that convert energy (ideally).
- **Modulated Transformers and Gyrators**: (MTF, MGY) elements that convert energy (driven by signals).
- **Other elements**: other elements and user defined elements.
Orientation

The orientation of a bond, i.e. the direction of the half arrow, denotes the direction of positive power. An element with an incoming bond consumes power when the product of effort and flow is positive. For R, C and I-elements an incoming bond is the standard orientation. For sources the standard orientation is outgoing, because a source supplies power to the rest of the system. For the TF and GY-element the standard orientation is one incoming bond and one outgoing bond, since this reflects the natural flow of power. For the other bonds in a model, as much as possible an orientation from source to load is applied.
Bonds and Signals

Bonds Graph models are useful when describing systems with powerflow. In mechatronic systems these systems are usually coupled to systems that handle (powerless) signals. These systems are more conveniently described by block diagrams. Bond graphs can be combined with block diagrams. The coupling can be performed by special submodels.

Sensors

In general a sensor measures the flow, the effort, the integral of the flow or the integral of the effort. Sensors can be found in the 20-sim bond graph library.

![Sensors](image)

Junctions

In 20-sim 1 junctions have a signal output which is equal to the flow. From a 1 junction a signal can be drawn which can be used as input for a block diagram. In a similar way an effort signal can be drawn from a 0 junction.

![Junctions](image)

Modulated Elements

The result of a block diagram can be converted into power by means of a generator. In a bond graph model, this can be done by connecting a signal to modulated source elements. These elements convert input signals into efforts or flows. Other elements that use a signal input are modulated transformers and modulated gyrators.

Example

An example model where a bond graph and a block diagram are coupled is shown in the picture below. As can be seen an effort signal is measured at the 0 junction. In the block diagram part it is processed and fed back into the bond graph at the modulated effort source.
Simplification of Bond Graph Models

When a bond graph model has been created by converting all elements of the iconic diagram into bond graph elements and connecting the elements, simplifications can be performed. This can be easily done by applying the following set of rules (the bonds that have no half arrow, are allowed in both orientations).

1. Eliminate loose junctions.

2. Eliminate junctions.

3. Melt equal junctions.

4. Eliminate sources with a zero output.

5. Eliminate junction in combination with a sign change on the source element.

6. Eliminate a double difference (two 0-junctions coupled with two 1-junctions)
Note

Simplification rules 1, 2, 3 and 6 can be performed automatically in 20-sim using the Simplify Model command.
Causality

Causal analysis is the determination of the direction of the efforts and flows in a bond graph model. The result is a causal bond graph which can be considered as a compact block diagram. From causal bond graph we can directly derive an equivalent block diagram. In 20-sim causality is assigned automatically. To create a causal bond graph 20-sim will perform the following steps.

1. Apply causality for all elements with fixed causality (Se,Sf,Mse,MSf, user defined models), i.e. assign a causal stroke to all bonds connected to elements with fixed causality.
2. Apply, as much as possible, causality for elements that have a constraint causality (0 junction, 1 junction, TF, GY, MTF, MGY, user defined models).
3. Apply causality for an arbitrary element with preferred causality (C,I, user defined models). If possible, let this be the preferred causality. Now iterate step 2 and 3 as far as possible.
4. Apply causality for an arbitrary element with indifferent causality (R, user defined models). If this causality is not fixed, choose a causality arbitrarily. Now iterate step 2, 3 and 4 as far as possible.

In many cases, the bond graph model is causal after step 2. However, sometimes as causal conflict occurs. This means the model is not correct arithmetically. Often this points out an ill-defined model. Redefinition of the ideal physical model will solve the problem. Sometimes the model is only causal after step 4. This means the model contains an algebraic loop. Models with algebraic loops are generally hard to simulate.

Example

Causal assignment will be illustrated by an example. We will manually create a causal bond graph out of the following model:

\[ R: \text{Re} \quad L: \text{L} \quad I: \text{J} \quad R: \dot{c} \]
\[ \text{Se}: u_m \quad 1 \quad \text{GY} \quad 1 \quad \omega \]

1. Apply fixed causality:

\[ R: \text{Re} \quad L: \text{L} \quad I: \text{J} \quad R: \dot{c} \]
\[ \text{Se}: u_m \quad 1 \quad \text{GY} \quad 1 \quad \omega \]

2. Apply constraint causality. This step is not yet applicable.

3. Apply preferred causality. We can choose between the two I-elements. Let’s select the left one and assign causality in its preferred form:

\[ R: \text{Re} \quad L: \text{L} \quad I: \text{J} \quad R: \dot{c} \]
\[ \text{Se}: u_m \quad 1 \quad \text{GY} \quad 1 \quad \omega \]

4. Apply constraint causality. The 1 junction on the left has one bond with effort-out causality (seen from the 1 junction). The other bonds connected must therefore have effort-in causality (seen from the 1 junction). In a same way we can now assign causality for the GY-element:
5. Apply preferred causality. Only the right I-element is left. We can assign causality in its preferred form:

6. Apply constraint causality. The 1 junction on the right has one bond with effort-out causality (seen from the 1 junction). The other bonds connected must therefore have effort-in causality. Therefore we can assign causality for the R-element:
**Iconic Diagrams to Bond Graphs (Electrical Domain)**

In the electrical domain, terminals and knots can be replaced by 0 junctions. With this knowledge and the description of various icons a table can be created that shows icons of electrical elements and their equivalent bond graph elements. The method of creating a bond graph model is:

1. Create an iconic diagram of the electrical system.
2. Reference points or grounds are split up as much as possible.
3. Replace every element of the diagram by its bond graph equivalent using the conversion table.
4. Replace every knot that has been left by a 0 junction.
5. Connect all elements and junctions with bonds according to the layout of the electrical system.
6. Simplify the resulting bond graph model, using the given set of *simplification rules*.

**Example**

The method will be illustrated by an example. We will convert an electrical circuit into a bond graph model.

1. Create an iconic diagram

   ![Iconic Diagram](image)

2. Split up reference points.

   ![Reference Points](image)

3. Replace elements by bond graph equivalents. Note that we have arranged the orientation of the bonds (the direction of the half arrow) as much as possible in the direction of the power flow from the source to the load elements.

   ![Bond Graph](image)
4. Replace every knot that has been left by a 0 junction. In this model no knots have been left. Step 4 is therefore not applicable.

5. Connect all elements and junctions with bonds, according to the layout of the electrical system. A comparison between the iconic diagram and the bond graph model clearly shows the symmetry between both representations. It is obvious that any change or addition in the iconic diagram, can be easily implemented in the bond graph model.

7. Simplify the resulting bond graph model. First we eliminate the Sources (Se elements) with zero output.

8. Eliminate loose junctions.


The simplified model does not show a direct symmetry with the iconic diagram. It does give a clear insight in the flows of power from the voltage source to the other elements.
**Iconic Diagrams to Bond Graphs (Mechanical Domain)**

In the mechanical domain, both translational and rotational, terminals and knots can be replaced by 1 junctions. With this knowledge and the description of various icons a table can be created that shows icons of mechanical elements and their equivalent bond graph elements. The method of creating a bond graph model is:

1. Create an iconic diagram of the mechanical system.
2. Reference points or grounds are split up as much as possible.
3. Replace every element of the diagram by its bond graph equivalent using the conversion table.
4. Replace every knot that has been left by a 1 junction.
5. Connect all elements and junctions with bonds, according to the layout of the mechanical system.
6. Simplify the resulting bond graph model, using the given set of simplification rules.

**Example**

The method will be illustrated by an example. We will convert an mechanical structure (translational) into a bond graph model.

1. Create an iconic diagram

![Iconic Diagram](image)

2. Split up reference points. In this diagram this is not possible.
3. Replace elements by bond graph equivalents. Note that we have arranged the orientation of the bonds (the direction of the harpoon) as much as possible in the direction of the power flow from the source to the load elements.

![Bond Graph](image)

4. Replace every knot that has been left by a 1 junction.
5. Connect all elements and junctions with bonds, according to the layout of the mechanical system. A comparison between the iconic diagram and the bond graph model clearly shows the symmetry between both representations. It is obvious that any change or addition in the iconic diagram, can be easily implemented in the bond graph model.

6. Simplify the resulting bond graph model. First we eliminate the Sources (Se elements) with zero output.

7. Eliminate loose junctions.
8. Eliminate junctions.

The simplified model does not show a direct symmetry with the iconic diagram. It does give a clear insight in the flows of power from the force source to the other elements.
From Bond Graph to Block Diagram

With a causal bond graph model, equivalent block diagram models can easily be derived. To create a block diagram, the following steps have to be performed.

- Change bonds by equivalent bilateral signals.

- Replace elements by corresponding block diagram symbols. Use the correct effort and flow description which can be found in the element tables. As an example both descriptions for the R-element are given.

- Replace the junctions by signal summation points and signal splitters. If correct, the bonds have already be replaced by effort and flow signals. Out of these signals and the junction description you can derive the effort and flow equations for the junction. As an example the conversion for a 1 junction is shown.

Example

As an example in the figure below a causal bond graph model is shown.

Using the given set of rules and the element descriptions an equivalent block diagram models is found, which is shown below.
The resulting block diagram model can be simplified by combining blocks and elimination of loops. Out of the block diagram, easily a set of dynamic equations can be deduced.
From Bond Graph to Equations

With a causal bond graph model, equivalent dynamic equations can easily be derived. To create the dynamic equations, the following steps have to be performed.

- Denote for every bond its effort and flow pair. You can use names that are obvious or methodically use numbers. Some examples are shown at the right.

- Replace elements by corresponding dynamic equations. Use the correct effort and flow description which can be found in the element tables. As an example descriptions for the R-element are given.

- Replace the junctions by the correct equations. For 0 junctions the efforts are equal. All flows of the bonds pointing towards the 0 junction should be added and all flows of the bonds pointing from the 0 junction should be subtracted. For 1 junctions the flows are equal. All efforts of the bonds pointing towards the 0 junction should be added and all efforts of the bonds pointing from the 0 junction should be subtracted. Some examples are shown at the right.

- Rearrange the equations by removing redundant variables.

**Example**

As an example in the figure below a causal bond graph model is shown. We will derive the set of dynamic equations out of this model.

1. Denote all efforts and flows. This is shown below:
2. Write the dynamic equations of the elements and junctions:

<table>
<thead>
<tr>
<th>Elements</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Se</td>
<td>$u_{in} = \text{constant}$, $i_{in} = \text{free}$</td>
</tr>
<tr>
<td>R</td>
<td>$u_r = Re i_r$</td>
</tr>
<tr>
<td>I</td>
<td>$i_j = (1/L) \int(u_j)$</td>
</tr>
<tr>
<td>1 Junction</td>
<td>$i_{in} = i_r = i_l = i_m$</td>
</tr>
<tr>
<td>GY</td>
<td>$u_m = k_m \omega_m$</td>
</tr>
<tr>
<td></td>
<td>$T_m = k_m i_m$</td>
</tr>
<tr>
<td>I</td>
<td>$T_j = (1/J) \int(\omega_j)$</td>
</tr>
<tr>
<td>R</td>
<td>$T_r = d \omega_r$</td>
</tr>
</tbody>
</table>

3. Reduce the amount of equations. We replace the equalities of the 1 junctions: $i_{in} = i_r = i_l = i_m = i$ and $\omega_m = \omega_j = \omega_r = \omega$.

<table>
<thead>
<tr>
<th>Elements</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Se</td>
<td>$u_{in} = \text{constant}$</td>
</tr>
<tr>
<td>R</td>
<td>$u_r = Re i$</td>
</tr>
<tr>
<td>I</td>
<td>$i_j = (1/L) \int(u_j)$</td>
</tr>
<tr>
<td>1 Junction</td>
<td>$u_{in} - u_r - u_l - u_m = 0$</td>
</tr>
<tr>
<td>GY</td>
<td>$u_m = k_m \omega$</td>
</tr>
<tr>
<td></td>
<td>$T_m = k_m i$</td>
</tr>
<tr>
<td>I</td>
<td>$T_j = (1/J) \int(\omega)$</td>
</tr>
<tr>
<td>R</td>
<td>$T_r = d \omega$</td>
</tr>
</tbody>
</table>

The resulting dynamic equations can be used for simulation. 20-sim can automatically extract the dynamic equations out of a bond graph model. The equations can be shown using the Show Equations command or used in the Simulator for simulation.
**Ports**

A port is a location where an element can exchange information (in case of a signal port) or power (in case of a power port) with its environment. So, it is the model port that defines the connection with the element. A port is an important concept, as it allows you to describe the properties of the bonds that can be connected to the element, i.e., its direction, size, domain, etc. Ports can be defined in 20-sim using the *Interface Editor*.

**Port Variables**

All ports of a submodel are shown in the *Interface tab*. In the figure below a standard *C-element* is shown with one power port $p$. Each port has an *effort and a flow* variable. In 20-sim these variables are be denoted with the extensions .e and .f. You can see an example in the figure below where equations are defined using the variables $p.e$ and $p.f$. 
Port Properties
Ports can be added and defined in the 20-sim Interface Editor. The Interface Editor of the C-element is shown below.
Bond graph ports have several properties:

- **Name**: The name of the port.
- **Type**: Next to bond graph ports, 20-sim also knows iconic diagram ports and signal ports.
- **Orientation**: The orientation of a connected bond (indicated by the half arrow).
  - **fixed in orientation**: The bond will point towards the element.
  - **fixed out orientation**: The bond will point from the element to another element.
- **Rows/Columns**: The standard size of a port and corresponding bond is 1 but you can also define ports with larger sizes.
- **Domain**: The domain of the port.
- **Causality**: The preferred causality of the port variable (effort and flow). You have to defined here what should be the input variable (effort or flow) and what should be the output variable (effort or flow).
**0 and 1 junctions**

Junctions couple one or more elements of a model in a power continuous mode: no energy is stored or dissipated. Examples are a series junction or parallel junction in an electrical network, a fixed connection between two mechanical parts etc. Two types of junctions exist: 0 junctions and 1 junctions.

**0 junction**

\[
\begin{align*}
&\text{\( e_1 \)} & & \text{\( f_1 \)} \\
&\text{\( f_2 \)} & & \text{\( e_2 \)} \\
&\text{\( f_3 \)} & & \text{\( e_3 \)} \\
&\text{\( e_1 = e_2 = e_3 \)} \\
&\text{\( f_1 - f_2 - f_3 = 0 \)}
\end{align*}
\]

The 0 junction represents a coupling where all efforts of the connected bonds are equal. As a consequence of the property of **power continuity** the sum of the flows must be equal to zero. The orientation of the bonds determines the **sign of the flow summation**: all flows of the bonds pointing towards the 0 junction should be added and all flows of the bonds pointing from the 0 junction should be subtracted. This summation corresponds to the **Kirchhoff current law** for electrical networks. The equality of the efforts, **limits the causality**. Only one bond may have an "effort-in" causality (the stroke pointed towards the 0 junction). All other bonds must have (seen from the 0 junction) an "effort-out" causality (stroke pointing from the 0 junction). In other words: the 0 junction has a **constraint causality**.

**1 junction**

\[
\begin{align*}
&\text{\( e_1 \)} & & \text{\( f_1 \)} \\
&\text{\( f_2 \)} & & \text{\( e_2 \)} \\
&\text{\( f_3 \)} & & \text{\( e_3 \)} \\
&\text{\( f_1 = f_2 = f_3 \)} \\
&\text{\( e_1 - e_2 - e_3 = 0 \)}
\end{align*}
\]

The 1 junction is the dual form of the 0 junction (effort and flow are opposite). The 1 junction represents coupling where all flows of the connected bonds are equal. As a consequence of the property of **power continuity** the sum of the efforts must be equal to zero. The orientation of the bonds determines the **sign of the flow summation**: all efforts of the bonds pointing towards the 1 junction should be added and all efforts of the bonds pointing from the 1 junction should be subtracted. This summation corresponds to the **Kirchhoff voltage law** for electrical networks. The equality of the flows, **limits the causality**. Only one bond may have an "effort-out" causality (the stroke pointing from the 1 junction). All other bonds must have (seen from the 1 junction) an "effort-in" causality (stroke pointing to the 1 junction). In other words: the 1 junction has a **constrained causality**.
0 junction

Suppose we have the following 0 junction with one bond (1) pointing towards the junction and two bonds (2 and 3) pointing from the junction.

\[ e_1 = e_2 = e_3 \]
\[ f_1 - f_2 - f_3 = 0 \]

1. For a 0 junction the efforts are always equal! This means:
\[ e_1 = e_2 = e_3 \]

2. The junction is power continuous. For the figure above this means:
\[ e_1 f_1 = e_2 f_2 + e_2 f_2 \]

3. Combining 1 and 2 yields:
\[ f_1 - f_2 - f_3 = 0 \]

The last equation can also be derived with a rule of thumb: All flows of the bonds pointing towards the 0 junction should be added and all flows of the bonds pointing from the 0 junction should be subtracted.

The first equation, limits the causality. Only one bond may have an effort-in causality (the stroke pointed towards the 0 junction). All other bonds must have (seen from the 0 junction) an effort-out causality (strokes pointing from the 0 junction). For the example junction this means three possible causal forms can exist:

\[ e_1 = e_2 = e_3 \]
\[ f_1 = f_2 + f_3 \]
\[ f_2 = f_1 - f_3 \]
1 junction

Suppose we have the following 1 junction with one bond (1) pointing towards the junction and two bonds (2 and 3) pointing from the junction.

1. For a 1 junction the flows are always equal! This means:

\[ f_1 = f_2 = f_3 \]

2. The junction is power continuous. For the figure above this means:

\[ e_1 f_1 = e_2 f_2 + e_2 f_2 \]

3. Combining 1 and 2 yields:

\[ e_1 - e_2 - e_3 = 0 \]

The last equation can also be derived with the rule of thumb: All effort of the bonds pointing towards a 1 junction should be added and all efforts of the bonds pointing from the 1 junction should be subtracted.

The first equation, limits the causality. Only one bond may have an flow-in causality (the stroke pointed from the 1 junction). All other bonds must have (seen from the 1 junction) a flow-out causality (strokes pointed towards the 1 junction). For the example junction this means three possible causal forms can exist:

\[
\begin{align*}
&\begin{array}{c}
\text{e}_1 \\
\text{f}_1
\end{array} & \begin{array}{c}
\text{e}_2 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_2 \\
\text{1}
\end{array} & \begin{array}{c}
\text{e}_3 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_3
\end{array} \\
& f_1 = f_2 = f_3 \\
& e_1 = e_2 + e_3
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
\text{e}_1 \\
\text{f}_1
\end{array} & \begin{array}{c}
\text{e}_2 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_2 \\
\text{1}
\end{array} & \begin{array}{c}
\text{e}_3 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_3
\end{array} \\
& f_1 = f_3 = f_2 \\
& e_2 = e_1 - e_3
\end{align*}
\]

\[
\begin{align*}
&\begin{array}{c}
\text{e}_1 \\
\text{f}_1
\end{array} & \begin{array}{c}
\text{e}_2 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_2 \\
\text{1}
\end{array} & \begin{array}{c}
\text{e}_3 \\
\uparrow
\end{array} & \begin{array}{c}
\text{f}_3
\end{array} \\
& f_1 = f_2 = f_3 \\
& e_3 = e_1 - e_2
\end{align*}
\]
Buffers

Buffers are bond graph elements that can store energy. There are two types of buffers: \textit{C-elements} and \textit{I-elements}. The table below shows the effort and flow descriptions that belong to these elements. The parameters "C" and "I" are the \textit{bufferconstants}, which determine a linear buffer behavior. Examples of C-elements are a mechanical spring and an electrical capacitor. Examples of an I-element are a mechanical inertia and an electrical inductance.

\begin{center}
\begin{tabular}{|l|c|c|}
\hline
\textbf{C-element} & \textbf{bond graph} & \textbf{equation} \\
& \textbf{element} & \\
\hline
effort-in causality & \begin{array}{c}
\dot{e} \\
\int f \\
C \\
\end{array} & f = C \frac{de}{dt} \\
\hline
effort-out causality & \begin{array}{c}
\int e \\
f \\
\end{array} & e = \frac{1}{C} \int f \\
\hline
\textbf{I-element} & \textbf{bond graph} & \textbf{equation} \\
& \textbf{element} & \\
\hline
effort-in causality & \begin{array}{c}
\int e \\
f \\
I \\
\end{array} & f = \frac{1}{I} \int e \\
\hline
effort-out causality & \begin{array}{c}
\dot{e} \\
\int f \\
I \\
\end{array} & e = I \frac{df}{dt} \\
\hline
\end{tabular}
\end{center}

Buffer elements do not fix the direction of the effort and flow. Both effort-in as well as effort-out causality is allowed. With simulation however, we prefer to avoid differentiation. In other words, with the C-element the effort-out causality is \textit{preferred} and with the I-element the effort-in causality is \textit{preferred}.
**Resistance**

A resistance, $R$, dissipates free energy. This energy of any arbitrary domain is transported *irreversibly* to the thermal domain. This means the power towards a resistance is always positive. In the table below the effort and flow description of the resistance is shown. Examples of a resistance are a mechanical damper and an electrical resistor.

<table>
<thead>
<tr>
<th>R-element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>effort-in causality</td>
<td><img src="image" alt="Diagram" /> $f = \frac{1}{R} e$</td>
<td></td>
</tr>
<tr>
<td>effort-out causality</td>
<td><img src="image" alt="Diagram" /> $e = R f$</td>
<td></td>
</tr>
</tbody>
</table>

The direction of effort and flow is not restricted for a resistance and there is no preferred form for the equation. In other words: the R-element has an *indifferent causality*. 
Sources
Sources represent the interaction of a systems with its environment. There are two types of source elements: effort sources (Se) and flow sources (Sf).

In the table below the effort and flow description of the source elements is shown. Examples of a flow source are an electrical current source and a hydraulic pump that generates a constant flow of liquid. Examples of an effort source are an electrical voltage source and a constant mechanical force.

<table>
<thead>
<tr>
<th>element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>effort source</td>
<td><img src="image" alt="Se" /></td>
<td>$e = c, f = \text{free}$</td>
</tr>
<tr>
<td>flow source</td>
<td><img src="image" alt="Sf" /></td>
<td>$f = c, e = \text{free}$</td>
</tr>
</tbody>
</table>

The direction of effort and flow for sources are restricted. In other words: the Se-element (effort-out) and Sf-element (effort-in) have a *fixed causality*. 
Modulated Sources

Next to the standard effort and flow sources, that generate a constant effort and flow, bond graph modeling also allows the use of so called modulated sources. In these sources the resulting effort or flow are equal to a (fluctuating) value provided by an input signal. Two types of modulated sources are known: the modulated effort source $MSe$ and the modulated flow source $MSf$. In the table below the effort and flow description of the source elements is shown.

<table>
<thead>
<tr>
<th>element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>modulated effort source</td>
<td>$MSe$</td>
<td>$e = u, f = \text{free}$</td>
</tr>
<tr>
<td>modulated flow source</td>
<td>$MSf$</td>
<td>$f = u, e = \text{free}$</td>
</tr>
</tbody>
</table>

The direction of effort and flow for sources are restricted. In other words: the $MSe$-element (effort-out) and $MSf$-element (effort-in) have a *fixed causality*. 
Transformers and Gyrators

Transformers and gyrators are bond graph elements that can convert energy ideally, as well in one physical domain as well as between one physical domain and another. A transformer is denoted by the mnemonic code TF and a gyrator by the code GY. In the table below the effort and flow equations are shown. The parameters r and n are the transformation ratio and gyration ratio. An examples of a transformer is a mechanical gear. An example of a gyrator is a DC-motor.

<table>
<thead>
<tr>
<th>TF-element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
</table>
| effort-in causality | \[ \frac{e_1}{f_1} \rightarrow \text{TF} \rightarrow \frac{e_2}{f_2} \] | \[ f_1 = \frac{1}{n} f_2 \]
|             |                   | \[ e_2 = \frac{1}{n} e_1 \] |
| effort-out causality | \[ \frac{e_1}{f_1} \rightarrow \text{TF} \rightarrow \frac{e_2}{f_2} \] | \[ f_2 = n f_2 \]
|             |                   | \[ e_1 = n e_2 \] |

<table>
<thead>
<tr>
<th>GY-element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
</table>
| effort-in causality | \[ \frac{e_1}{f_1} \rightarrow \text{GY} \rightarrow \frac{e_2}{f_2} \] | \[ f_1 = \frac{1}{r} e_2 \]
|             |                   | \[ f_2 = \frac{1}{r} e_1 \] |
| effort-in causality | \[ \frac{e_1}{f_1} \rightarrow \text{GY} \rightarrow \frac{e_2}{f_2} \] | \[ e_1 = r f_2 \]
|             |                   | \[ e_2 = r f_1 \] |

The directions of the flow and effort are partly fixed for a transformer. Like the 1 junction and the 0 junction, the causality is constraint. For a transformer, an effort-in causality on the incoming bond results in an effort-out causality on the outgoing bond and vice-versa. For a gyrator, an effort-in causality on the incoming bond results in an effort-in causality on the outgoing bond and vice-versa.
Modulated Transformers and Gyrators

Next to transformers and gyrators with a fixed transformation ratio and gyration ratio, in bond graphs also modulated transformers (MTF) and modulated gyrators (MGY) are supported. In these models the transformation ratio of gyration ratio are equal to a (fluctuating) value provided by an input signal. In the table below the effort and flow equations are shown.

<table>
<thead>
<tr>
<th>MTF-element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
</table>
| effort-in causality | ![MTF Effort-In](image) | \[ f_1 = \frac{1}{u} f_2 \]
|               |                    | \[ e_2 = \frac{1}{u} e_1 \] |
| effort-out causality | ![MTF Effort-Out](image) | \[ f_2 = u f_2 \]
|                       |                    | \[ e_1 = u e_2 \] |

<table>
<thead>
<tr>
<th>MGY-element</th>
<th>bond graph element</th>
<th>equation</th>
</tr>
</thead>
</table>
| effort-in causality | ![MGY Effort-In](image) | \[ f_1 = \frac{1}{u} e_2 \]
|               |                    | \[ f_2 = \frac{1}{u} e_1 \] |
| effort-in causality | ![MGY Effort-In](image) | \[ e_1 = u f_2 \]
|               |                    | \[ e_2 = u f_1 \] |

The directions of the flow and effort are partly fixed for a modulated transformer. Like the 1 junction and the 0 junction, the causality is constraint. For a transformer, an effort-in causality on the incoming bond results in an effort-out causality on the outgoing bond and vice-versa. For a gyrator, an effort-in causality on the incoming bond results in an effort-in causality on the outgoing bond and vice-versa.