Math 053 Calculus II. Fall 2008.

Solutions to Special Project from Homework 4

Statement of the problem: Compute the volume of the Atchley clock tower (small tower) on campus.

Solutions

To measure a tall building or structure without having its blueprint is a challenge that every engineer has to deal with at some point. There are several ways to do this, some involving the use of theodolites or total stations that are very advanced technological tools that land surveyors, engineers and other professionals use every day. The students did not have access to those tools and so they had to come up with alternative ways to measure the height of the Atchley tower. There were basically 3 different techniques that came up in the discussions:

1. Fold a paper like a triangle with a 45 degree angle and hold it at a certain distance from the tower. Look along the side of the paper until you visualize the top of the tower. Measure the distance from your feet to your eyes and the distance from your feet to the base of the tower and use the trigonometry of a right triangle to compute the height. This was the method used by Joe and Nick from Section 01. The volume measured by Joe and Nick was $95.49 \text{ m}^3$.

2. Take a photograph of the tower and, using the measurement of some particular part of the structure that would be easy to measure, find the scale ratio between the actual structure and the picture. Use the picture to get the dimensions of the tower, then re-scale the dimensions and compute the volume. This was the method used by Andrew from Section 03. The volume measured by Andrew was $88.94 \text{ m}^3$. He worked with AutoCAD (a very powerful design tool used in all areas of engineering, architecture, etc.). See Figure 2.

3. Finally, taking advantage of the blocks by which the tower is made one can get an approximation to its height by measuring one block and counting (or estimating) how many blocks cover its height.

Notice also that by looking at the tower (and looking at the people going by around of it) we can get a basic approximation to the volume. The tower is more or less like a shoe box of about $3\text{m} \times 3\text{m} \times 10\text{m} \approx 90\text{m}^3$. Of course, this is a rough estimate and the actual volume will be different from this number. The average of both measurements is about $92\text{m}^3$. This will be the official volume of the Atchley tower until new measurements are made!

An interesting feature of this tower is the fact that the arc on the base is a semicircle (see Figure 1). If you study civil engineering (or take some courses on structural design) you will learn that the most efficient form in arcs is the parabola not the circle. If you look
at how bridges are designed or at the interior of a cathedral you will find parabolic arcs sustaining the main structure and ceilings\textsuperscript{1}.

Joe, Nick and Andrew, you did an excellent job. Congratulations!

\begin{figure}[h]
\centering
\includegraphics{AtchleyClockTowerBase.png}
\caption{The base of the Atchley clock tower.}
\end{figure}

\textsuperscript{1}The Atchley clock tower was made in this way probably because it is a small structure that did not required the slightly more complicated parabolic design.
Figure 2: The Atchley clock tower. AutoCAD drawings and photograph.