Section 1.3

Rates of Change
If \( y \) is a function of \( t \), so \( y = f(t) \), then

\[
\text{Average rate of change of } y \text{ between } t = a \text{ and } t = b = \frac{\Delta y}{\Delta t} = \frac{f(b) - f(a)}{b - a}.
\]

The units of average rate of change of a function are units of \( y \) per unit of \( t \).

**Figure 1.26**

**Figure 1.27**
11. Table 1.10 shows the production of tobacco in the US.\textsuperscript{19}

(a) What is the average rate of change in tobacco production between 1996 and 2003? Give units and interpret your answer in terms of tobacco production.

(b) During this seven-year period, is there any interval during which the average rate of change was positive? If so, when?

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
\hline
Production & 1517 & 1787 & 1480 & 1293 & 1053 & 991 & 879 & 831 \\
\hline
\end{tabular}
\caption{Tobacco production, in millions of pounds}
\end{table}
A function $f$ is **increasing** if the values of $f(x)$ increase as $x$ increases. A function $f$ is **decreasing** if the values of $f(x)$ decrease as $x$ increases.

The graph of an increasing function **climbs** as we move from left to right. The graph of a decreasing function **descends** as we move from left to right.

The graph of a function is **concave up** if it bends upward as we move left to right; the graph is **concave down** if it bends downward. (See Figure 1.29.) A line is neither concave up nor concave down.
6. Identify the $x$-intervals on which the function graphed in Figure 1.33 is:

(a) Increasing and concave up
(b) Increasing and concave down
(c) Decreasing and concave up
(d) Decreasing and concave down

Problem 6
5. Table 1.9 gives values of a function $w = f(t)$. Is this function increasing or decreasing? Is the graph of this function concave up or concave down?

Table 1.9

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
<td>100</td>
<td>58</td>
<td>32</td>
<td>24</td>
<td>20</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>
Section 1.4

Applications of Functions to Economics
The **cost function**, $C(q)$, gives the total cost of producing a quantity $q$ of some good.

If $C(q)$ is a linear cost function,
- Fixed costs are represented by the vertical intercept.
- Variable cost per unit is represented by the slope.

The **revenue function**, $R(q)$, gives the total revenue received by a firm from selling a quantity, $q$, of some good.
Profit = Revenue - Cost so $\pi = R - C$.

7. Figure 1.54 shows cost and revenue for a company.

(a) Approximately what quantity does this company have to produce to make a profit?
(b) Estimate the profit generated by 600 units.
The **supply curve**, for a given item, relates the quantity, \( q \), of the item that manufacturers are willing to make per unit time to the price, \( p \), for which the item can be sold. The **demand curve** relates the quantity, \( q \), of an item demanded by consumers per unit time to the price, \( p \), of the item.

**Figure 1.47**
Figure 1.48
• 1.4: 2, 3, 4, 5, 9, 19