Instantaneous Rate of Change
We throw a grapefruit straight upward into the air. Table 2.1 gives its height, $y$, at time $t$. What is the velocity of the grapefruit at exactly $t = 1$? We use average velocities to estimate this quantity.

**Table 2.1  ** *Height of the grapefruit above the ground*

<table>
<thead>
<tr>
<th>$t$ (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = s(t)$ (feet)</td>
<td>6</td>
<td>90</td>
<td>142</td>
<td>162</td>
<td>150</td>
<td>106</td>
<td>30</td>
</tr>
</tbody>
</table>

Average velocity between $t = 1$ and $t = 1.01$

$$\Delta v = \frac{\Delta y}{\Delta t} = \frac{s(1.01) - s(1)}{1.01 - 1} = \frac{90.678 - 90}{0.01} = 67.8 \text{ ft/sec}.$$
<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>0.9</th>
<th>0.99</th>
<th>0.999</th>
<th>1</th>
<th>1.001</th>
<th>1.01</th>
<th>1.1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = s(t)$</td>
<td>6.000</td>
<td>83.040</td>
<td>89.318</td>
<td>89.932</td>
<td>90.000</td>
<td>90.068</td>
<td>90.678</td>
<td>96.640</td>
<td>142.000</td>
</tr>
</tbody>
</table>

- Average velocity: 84 ft/sec
- Average velocity: 52 ft/sec
- Average velocity: 69.6 ft/sec
- Average velocity: 66.4 ft/sec
- Average velocity: 68.2 ft/sec
- Average velocity: 67.8 ft/sec
- Average velocity: 68.0 ft/sec
- Average velocity: 68.0 ft/sec

**Figure 2.1**
The **instantaneous velocity** of an object at time $t$ is defined to be the limit of the average velocity of the object over shorter and shorter time intervals containing $t$.

The **instantaneous rate of change** of $f$ at $a$, also called the **rate of change** of $f$ at $a$, is defined to be the limit of the average rates of change of $f$ over shorter and shorter intervals around $a$. 


Example

Let \( f(x) = 0.15x^2 \).

1. Calculate the average rate of change in \( f \) over the interval from \( x = 2 \) to \( x = 5 \).

2. Estimate the instantaneous rate of change in \( f \) at \( x = 2 \) using intervals that get shorter and shorter around \( x = 2 \).
The derivative of $f$ at $a$, written $f'(a)$, is defined to be the instantaneous rate of change of $f$ at the point $a$.

**Figure 2.2**

The derivative of a function at the point $A$ is equal to

- The slope of the graph of the function at $A$.
- The slope of the line tangent to the curve at $A$.

**Figure 2.3**
9. (a) The function \( f \) is given in Figure 2.13. At which of
the labeled points is \( f'(x) \) positive? Negative? Zero?
(b) At which labeled point is \( f' \) largest? At which la-
beled point is \( f' \) most negative?
17. Table 2.3 gives $P = f(t)$, the percent of households in the US with cable television $t$ years since 1990.³

(a) Does $f'(6)$ appear to be positive or negative? What does this tell you about the percent of households with cable television?

(b) Estimate $f'(2)$. Estimate $f'(10)$. Explain what each is telling you, in terms of cable television.

<table>
<thead>
<tr>
<th>$t$ (years since 1990)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$ (% with cable)</td>
<td>59.0</td>
<td>61.5</td>
<td>63.4</td>
<td>66.7</td>
<td>67.4</td>
<td>67.8</td>
<td>68.9</td>
</tr>
</tbody>
</table>

Problem 17
The table gives fictional data about farm land in the Central Valley. \( A \) is the number of acres of farm land (in millions) as a function of \( t \), years since 1980.

<table>
<thead>
<tr>
<th>( t ), years since 1980</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>acres of farm land (in millions)</td>
<td>1000</td>
<td>960</td>
<td>930</td>
<td>910</td>
<td>900</td>
</tr>
</tbody>
</table>

1. Is the derivative positive or negative?
2. Estimate the \( A'(20) \).