Section 4.1

Local Maxima and Minima
A local maximum is a point with $y$ value greater than the other $y$ values of all other points in an interval around it.

A local minimum is a point with $y$ value less than the $y$ values of all other points in an interval around it.

The derivative is 0 at local maximum and local minimum points.

A critical point of a function $f$ is a point where $f'(x) = 0$. 
First Derivative Test for Local Maxima and Minima

Suppose $p$ is a critical point of a continuous function $f$.

- If $f$ changes from decreasing to increasing at $p$, then $f$ has a local minimum at $p$.
- If $f$ changes from increasing to decreasing at $p$, then $f$ has a local maximum at $p$.

Second Derivative Test for Local Maxima and Minima

Suppose $p$ is a critical point of a continuous function $f$, and $f'(p) = 0$.

- If $f$ is concave up at $p$, then $f$ has a local minimum at $p$.
- If $f$ is concave down at $p$, then $f$ has a local maximum at $p$. 
Let $f(x) = x^3 - 9x^2 - 48x + 52$.

- Find the local maxima and local minima.
- Graph $f$ with your calculator, finding a viewing window that shows the maxima and minima.
Suppose \( p \) is a point in the domain of \( f \):

- \( f \) has a local minimum at \( p \) if \( f(p) \) is less than or equal to the values of \( f \) for points near \( p \).
- \( f \) has a local maximum at \( p \) if \( f(p) \) is greater than or equal to the values of \( f \) for points near \( p \).
Let $f(x) = x^4 - 4x^3 + 10$.

- Find the local maxima and local minima.
- Graph $f$ with your calculator, finding a viewing window that shows the maxima and minima.
7. Graph two continuous functions \( f \) and \( g \), each of which has exactly five critical points, the points \( A-E \) in Figure 4.12, and which satisfy the following conditions:

(a) \( f(x) \to \infty \) as \( x \to -\infty \) and \\
\[ f(x) \to \infty \text{ as } x \to \infty \]

(b) \( g(x) \to -\infty \) as \( x \to -\infty \) and \\
\[ g(x) \to 0 \text{ as } x \to \infty \]
28. Assume $f$ has a derivative everywhere and has just one critical point, at $x = 3$. In parts (a)–(d), you are given additional conditions. In each case decide whether $x = 3$ is a local maximum, a local minimum, or neither. Explain your reasoning. Sketch possible graphs for all four cases.

(a) $f'(1) = 3$ and $f'(5) = -1$
(b) $f(x) \to \infty$ as $x \to \infty$ and as $x \to -\infty$
(c) $f(1) = 1, f(2) = 2, f(4) = 4, f(5) = 5$
(d) $f'(2) = -1, f(3) = 1, f(x) \to 3$ as $x \to \infty$

Problem 28
Section 4.2

Inflection Points
A point at which the graph of a function \( f \) changes concavity is called an \textbf{inflection point} of \( f \).

\[ f''(p) = 0 \]

Point of inflection

\[ f'' > 0 \]

Concave up

\[ f'' < 0 \]

Concave down

\[ f''(p) = 0 \]

Point of inflection

\[ f'' > 0 \]

Concave up

\[ p \]

\[ p \]

\textbf{Figure 4.16}
Let \( f(x) = x^3 - 9x^2 - 48x + 52 \).

Find the point(s) of inflection.

Let \( f(x) = x^4 - 4x^3 + 10 \).

Find the point(s) of inflection.